Fermi National Accelerator Laboratory

FERMILAB-PUB-86/113-T
August, 1986

Ginzburg-Landau Type Effective Theory for Deconfinement and Chiral Transitions in QCD

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Abstract

We construct an effective potential in power series of the Wilson line and the chiral condensate order parameters. In a suitable range of numerical parameters, the potential qualitatively describes the deconfinement and chiral transitions, which take place at the same temperature, as indicated from the recent Monte-Carlo calculation of lattice *QCD* at finite temperature.

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Many people now believe that quarks and gluons are liberated and the chiral symmetry is restored at high temperatures while they are confined and the chiral symmetry is spontaneously broken at low temperature if the quark masses are ignored.

Recent developments[1][2] of Monte Carlo calculation in the lattice gauge theory seem to suggest that the deconfining and chiral transitions of QCD matter are first order and that the transition temperatures are almost the same, though there remain some controversies. Although we have not yet understood the underlying physics of the interrelation between the deconfining and chiral transitions, our intuition to QCD matter at finite temperature will be enhanced if we are able to construct an effective theory which can qualitatively (at least) reproduce the result of the Monte Carlo simulation.

The order parameter of deconfining transition[3] is the Wilson line,

$$\Omega = < TrPexp(i\int_0^{1/T} \lambda^a A_0^a d\tau) > = e^{-F_q/T} \tag{1}$$

with λ^a being the generator of $SU_c(3)$ in the fundamental representation. Here F_q is a free energy of a single static quark. In the absence of dynamical quarks, the nonzero expectation value Ω implies a spontaneously breaking of Z_3 symmetry, the center of color SU(3) group.[4] This also means that the quarks are deconfined, since the free energy of a single static quark F_q is finite. On the other hand, if $\Omega = 0$ the Z_3 symmetry is restored and the quarks are confined since F_q is then infinite. In the presence of dynamical quarks, the Z_3 symmetry is explicitly broken and the Wilson line is only an approximate order parameter. However, it turns out that Ω is a fairly good order parameter to describe the deconfining transition, since Ω is actually very small below the critical temperature. We will later discuss the effects of the explicit Z_3 breaking.

For the chiral symmetry, the order parameter is the chiral field matrix, or the quark condensate

$$M_{ij} = < \bar{\psi}_i(1+\gamma_5)\psi_j > \quad (i,j = 1,2,\cdots N_f) \ , \tag{2}$$

with i, j being flavor indices.

In order to make our discussion definite, we assume that the qualitative feature of the hysteresis curves is like the ones shown in Fig. 1. Similar hysteresis curves have been clearly observed in the recent numerical calculation with dynamical fermions by Fukugita and Ukawa.[2] One of the important points to notice is that the terminal temperatures T_1 and T_2 of the hysteresis appear to be common to both the deconfining order parameter $\phi = Re\Omega$ and the chiral condensate $\sigma = \sum_{i=1}^{N_f} < \bar{\psi}_i \psi_i >$. Below T_1 the deconfining order parameter $\phi = Re\Omega$ is very small and the system is in the confining phase. It remains small until the temperature goes up to T_2. Above T_2, ϕ jumps up to a value $O(1)$ and the quarks become deconfined. When the system cools down from a high temperature, the order parameter ϕ remains $O(1)$ when the temperature passes through T_2 and until it goes down to T_1. There the order parameter becomes very small again and the system goes back to the confining phase. We can do a similar description for the chiral order parameter σ which shows the chiral symmetry breaking and its restoration.

We are going to construct a Ginzburg-Landau type potential which is valid near the critical temperature and describes the qualitative feature of the phase transitions stated above.

Let us write down an effective potential $V(\Omega, M)$ in a power series of Ω and M up to forth order,

$$\begin{aligned} V &= V(\Omega, M) \\ &= \frac{a}{2}\Omega^+\Omega - \frac{b}{3}Re(\Omega^3) + \frac{c}{4}(\Omega^+\Omega)^2 \\ &\quad +\frac{\alpha}{2}(tr(M^+M))^2 + \frac{\gamma_1}{4}(tr(M^+M))^2 + \frac{\gamma_2}{4}tr(M^+M)^2) \\ &\quad +\frac{\beta}{3}Re(detM) \\ &\quad +\frac{\lambda}{2}\Omega^+\Omega tr(M^+M) \\ &\quad +V_{Z_{3\mathbf{breaking}}} \quad . \end{aligned} \tag{3}$$

This effective potential respects the chiral symmetry under $M \to UMV^+$, $U \epsilon SU_L(N_f)$, $V \epsilon SU_R(N_f)$.[5]* The terms except the last one are symmetric under $Z_3, \Omega \to z\Omega, z\epsilon Z_3$. There are a variety of Z_3 breaking terms, $Re\Omega, Re\Omega^2, Re\Omega^2\Omega^+$, $Im\Omega, \cdots Re\Omega \cdot tr(M^+M) Re\Omega^2 tr(M^+M) \cdots$.

Let us assume $Im\Omega = 0$ and retain only two degrees of freedom $\phi = Re\Omega$ and

$\sigma = trM/N_f$. For the purpose of comparison with the Monte Carlo calculation of Ref. [2], we consider the case $N_F = 4$. We end up with

$$\begin{aligned} V(\phi,\sigma) &= \frac{a}{2}\phi^2 - \frac{b}{3}\phi^3 + \frac{c}{4}\phi^4 \\ &+ \frac{\alpha}{2}\sigma^2 \quad + \frac{\gamma}{4}\sigma^4 \\ &+ \frac{\lambda}{2}\sigma^2\phi^2 + d\phi + k\phi\sigma^2 \ . \end{aligned} \tag{4}$$

(We have used the same symbols for the coefficients as in $Eq.$ (4), for notational simplicity).

Only the last two terms are genuine Z_3 breaking ones. The terms like $a/2 \cdot \phi^2$, $b/3 \cdot \phi^3, c/4 \cdot \phi^4$ and $\lambda/2 \cdot \sigma^2\phi^2$ may get some contribution from the Z_3 breaking terms. The terms in the first line also exist in the pure $SU_c(3)$ gauge system and the cubic term is responsible for the first order phase transition of confinement and deconfinement in that system.[4] As we shall see later, if we adopt the effective potential (4), the cubic term ϕ^3 is a driving force of the first order phase transition. The λ term in the last line gives an essential feature of the interplay of deconfining and chiral transitions observed in the numerical simulations. We parametrize the coeeficients of the quadratic terms as

$$\begin{aligned} a &= a_1(T_1' - T) \\ \alpha &= \alpha_1(T - T_2\prime) \end{aligned} \tag{5}$$

with $a_1, \alpha_1, T_1\prime$ and $T_2\prime$ being parameters. It is assumed that the coupling constants b, c, γ, k and λ are independent of temperature. The coefficient d of the linear term in ϕ may well depend on the temperature T. We have to caution ourselves that our assumption on the parameters in the effective potential may be a good one only near the critical temperature.

The analysis of the potential (4) is elementary. We first consider the case $a = k = d = 0$ in order to make our presentation as simple as possible. Later we shall give the results for nonvanishing a, k and d and discuss the effects of the Z_3 breaking.

[1]The authors of Ref. [5] considered the chiral model and obtained a so-called fluctuation induced first order phase transition for $N_F \geq 2$. Here our approach is completely different and much more naive.

The extrema of the potential are given by

$$\begin{aligned}\frac{\partial V}{\partial \phi} &= \phi(-b\phi + c\phi^2 + \lambda\sigma^2) = 0 \ , \\ \frac{\partial V}{\partial \sigma} &= \sigma(\alpha + \gamma\sigma^2 + \lambda\phi^2) = 0\end{aligned} \tag{6}$$

For a locally stable state, the Hessian matrix

$$H = \begin{bmatrix} \partial^2 V/\partial\phi^2, & \partial^2 V/\partial\phi\partial\sigma \\ \partial^2 V/\partial\sigma\partial\phi, & \partial^2 V/\partial\sigma^2 \end{bmatrix} \tag{7}$$

has to be positive definite. It turns out that the only candidates of local minima are

$$\left.\begin{aligned}(\phi_0, 0) \equiv & \ d \ \ \text{(deconfinement)} \\ (0, \sigma_0) \equiv & \ c \ \ \text{(confinement)}\end{aligned}\right\} \tag{8}$$

$$\left.\begin{aligned}\phi_0 &= b/c \\ \sigma_0 &= \sqrt{\tfrac{-\alpha}{\gamma}} = \sqrt{\tfrac{\alpha_1(T_2'-T)}{\gamma}}\end{aligned}\right\}; . \tag{9}$$

($(0, -\sigma_0)$ is equivalent to $(0, \sigma_0)$ due to the symmetry).

The other extrema are either local maxima or saddle points if $\lambda^2 < \gamma c$, which is a necessary inequality for the structual stability of V. We assume that $\alpha, b, c, \gamma, T_2'$ are positive.

At the deconfinement point $d = (\phi_0, 0)$ the value of the free energy and the Hessian are respectively

$$\begin{aligned}V_d &= -\frac{b}{12}\left(\frac{b}{c}\right)^3 \\ H_d &= \begin{bmatrix} b^2/c^2 & 0 \\ 0 & \alpha_1(T - T_2') + \lambda b^2/c^2 \end{bmatrix}\end{aligned} \tag{10}$$

The confinement point exists only if $T < T_2'$ and there the free energy and the Hessian are

$$\begin{aligned}V_c &= -\frac{\alpha_1^2}{4\gamma}(T - T_2')^2 \\ H_c &= \begin{pmatrix} \frac{\lambda}{\gamma}\alpha_1(T_2' - T) & 0 \\ 0 & 2\alpha_1(T_2' - T) \end{pmatrix} \ .\end{aligned} \tag{11}$$

It is easy to observe from Eqs.(10) and (11) that (A) for $T > T_2'$ d is the only local minimum and H_d is positive. Hence the deconfining and chiral symmetric phase is realized. (B) for $T_2' > T > T_1 \equiv T_2' - \frac{\lambda}{\alpha_1}(\frac{b}{c})^2$, both d and c are local minima. (c) for $T < T_1$, d becomes a saddle point, since one of the eigen values of H_d becomes negative. Here c is the only local minimum so that the confining and chiral symmetry broken phase is realized. The first order phase transition will take place through a tunneling at a point where $V_d = V_c$, or

$$T_{eq} = T_2' - \frac{1}{\alpha_1}\sqrt{\frac{c\gamma}{3}}\left(\frac{b}{c}\right)^2 \quad . \tag{12}$$

The obvious requirement $T_2 > T_{eq} > T_1$ implies the restriction to the coupling constant λ

$$\lambda > \sqrt{\frac{c\gamma}{3}} \quad . \tag{13}$$

Perhaps the conceptual picture, Fig. 2 may be helpful to understand what is going on in the transitin region. In summary, we have gotten hysteresis curves shown in Fig. 1 with $T_1 = T_2' - \frac{\lambda}{\alpha_1}(\frac{b}{c})^2$, $T_2 = T_2'$ $\phi_0 = b/c$ and $\sigma_0 = \sqrt{\alpha_1(T_2 - T)/\gamma}$. The latent heat associated with the first order phase transition at $T = T_{eq}$. is

$$L = T\frac{\partial V}{\partial T}|_{Teq-0} - T\frac{\partial V}{\partial T}|_{Teq+0} = \frac{\alpha_1^2}{2\gamma}T_{eq}\cdot(T_2' - T_{eq}) \quad . \tag{14}$$

This completes the analysis of the effective potential in the simplest case, $a = k = d = 0$.

For more general cases, we just quote the results. For a nonvanishing but a small a compared with b and c, $T_1 = T_2' - \frac{\lambda}{\alpha_1}(\frac{b}{c})^2, T_2 = (a_1T_1' + \lambda\frac{\alpha_1}{\gamma}T_2')/(a_1 + \lambda\frac{\alpha_1}{\gamma})$,

$$\begin{aligned} \phi_0 &= \frac{b + \sqrt{b^2 + 4a_1c(T - T_1')}}{2c} \quad , \\ \sigma_0 &= \sqrt{\frac{\alpha_1(T_2' - T)}{\gamma}} \quad , \end{aligned} \tag{15}$$

and

$$L = \frac{T_eq}{2}\left\{a_1(\frac{b}{c})^2 + \frac{\alpha_1}{\gamma}(T_2' - T_eq)\right\} \quad . \tag{16}$$

The expression for T_eq is given by $V_d = V_c$. Here we have assumed that $\alpha_1 > 0, T_2' > T_1'$, which implies the point $(\phi, \sigma = (0,0)$, namely confining and chiral symmetric phase is locally unstable for all T. We can easily check that there are only two candidates of local minima, $(\phi_0, 0)$ and $(0, \sigma_0)$, if $\alpha_1 > 0, T_2' > T_1'$.

Let us go back to Eq. (4) and take account of the linear terms in the Wilson line which are purely Z_3 breaking effects. The extrema are now given by

$$\begin{aligned}
\frac{\partial V}{\partial \phi} &= a\phi - b\phi^2 + c\phi^3 + \lambda\sigma^2\phi + d + k\sigma^2 = 0 \\
\frac{\partial V}{\partial \sigma} &= \alpha\sigma + \gamma\sigma^3 + \lambda\sigma\phi^2 = 0 \ .
\end{aligned} \tag{17}$$

Note that the Monte Carlo calculation shows that $\phi = Re\Omega$ is very small in the region of a finite σ. This implies that either d and k are very small or we need a fine tuning of parameters α, γ, d and k so that $d + k\sigma_0^2 \approx \alpha + \gamma\sigma_0^2 \approx 0$ hold simultaneously. We have already considered the first case. Let us consider the second case. The potential analysis similar to the previous one shows that we also have the first order phase transition with

$$\begin{aligned}
T_1 &= T_2' - \frac{\lambda}{\alpha_1}(\frac{b}{c})^2 \\
T_2 &= \frac{\lambda/\gamma\alpha_1 T_2' + a_1 T_1' - 2k^2/\gamma}{\lambda/\gamma\alpha_1 + a_1} \ .
\end{aligned} \tag{18}$$

The above expressions (17) shows that the hysteresis curve shrinks for a larger k, a Z_3 breaking coupling. In Ref [6], it is argued that for a large quark mass $m_q \gg T_{eq}$ the linear term $d\phi$ gives a main quark mass dependence of the effective potential and $d \sim exp(-m_q \times const)$. Since k is proportional to d from the fine tuning, this implies that the hysteresis is enhanced for a larger value of the quark mass m_q in the region $m_q \gg T_{eq}$. This quark mass dependence is consistent with the Monte Carlo results and also with the arguments given in Ref.s [6] and [7]. This is also intuitively understandable, since for a large quark mass the system approaches the pure $SU_c(3)$ gauge system. For a small quark mass $m_q \ll T_eq$, we do not know the quark mass dependences of k and d. If we assume that they are not significant and that the main quark mass dependence comes from the additional so-called σ term, $m\sigma$, we have

$$
\begin{aligned}
T_1 &= \frac{\lambda^2}{\alpha_1 c}\left(\frac{cm}{\lambda^2}\right)^{2/3} \\
&\quad +(\text{terms indenpendent of m}) \\
T_2 &= -\frac{\lambda^{1/2} m}{\alpha_1}\left(\alpha_1 (T_2' - T_1')\right)^{-1/2} \\
&\quad +(\text{terms independent of m}) \ . \qquad (19)
\end{aligned}
$$

We may easily recognize that for $m_q \ll T_{eq}$ or for a very small value of m, T_1 increases and T_2 decreases as m increases and hence the hysteresis curves will shrink. However we have to keep in mind that the issue of the quark mass dependence of the parameters of the effective potential is a highly dynamical one and therefore that the expressions (17) and (18) will be two of possibilities of the quark mass dependence of the hysteresis curves at best.

In summary we have constructed a simple Ginzburg-Landau type effective potential in terms of the Wilson line and the chiral condensate, which exhibits the first order phase transition of deconfining and chiral symmetry restoration characterized by the hysteresis curves in Fig. 1. At high temperature $T > T_2$, the quark-antiquark condensates dissapear, $\sigma = 0$ and the chiral symmetry is restored. The deconfining phase $\phi \approx b/c$ becomes energetically more favorable than the confining phase $\phi = 0$. In the intermediate temperature region $T_{eq} < T < T_2$, the deconfining and chiral symmetric (the confining and chiral symmetry broken) phase is a stable (metastable) state. For $T_1 < T < T_{eq}$, the other way round is realized. At low temperature $T < T_1$, the quark-antiquark pair condenses, $\sigma \neq 0$ and the chiral symmetry is spontaneously broken. This makes the confining phase $\phi = 0$ energetically more favorable than the deconfining one $\phi \neq 0$ owing to the term $\lambda\sigma^2\phi^2/2$.

The cubic term in ϕ in Eq. (4) is responsible for the first order phase transition. The interplay of the deconfining and the chiral transition can be explained by the term $\lambda\phi^2\sigma^2/2$ if $\sqrt{\gamma c/3} < \lambda < \sqrt{\gamma c}$. An attempt is also made to explain the observed quark mass dependence of the hysteresis.

It is easy to construct an effective potential which includes $Im\Omega$ and has a minimum at $Im\Omega = 0$. Our model can also be regarded as the one for $N_F = 2$, since in that case the determinant term in Eq.(3) is reduced to a quadratic term in σ. An extention to $N_F = 3$ needs a cubic term in σ, which makes the analysis slightly complicated.

In this paper, we have been concerned with the "local minima scenario" which automatically implies that the deconfining and chiral transitions take place at the same temperature, $T_d = T_{ch}$. If we allow more than three local minima, $(\phi_0, 0), (0,0)$ and $(0, \sigma_0)$ by choosing $a_1 < 0$, it is possible to construct a model in which the two transition temperatures are different, $T_d \neq T_{ch}$. This may be the case for the quarks in higher representations.[8] What we would like to emphasize here is that $T_d = T_{ch}$, if true, is not a numerical coincidence but rather a consequence of the global structure of the effective potential.

So far we have not discussed the possibility of microscopic derivation of our effective potential. Although it will be very difficult near the transition temperature region, some tendency may be observed by calculating the effective potential in the continuum perturbation theory at high temperature.[9] At low temperature we may use the hopping parameter expansion.[9],[10]

Although it seems premature, we can in principle fit our parameters to the Monte Carlo data if the data are given in terms of physical values. For example in the simplest version of our model the values of T_1, T_2, T_{eq}, the latent heat and the level-off value of $Re\Omega$ can determine all the parameters in the effective potential.

The author hopes that the Monte Carlo calculations of various groups converge and that the effective potential fitted to the numerical calculation data can serve the phenomenology of quark-gluon plasma.[11] After completion of this work, the author has received a preprint by Midorikawa, so and Yoshimoto.[12] They treated the same model by the renormalization group equation assuming that the coefficients of quardratic terms a and α vanish at the same temperature.

ACKNOWLEDGEMENTS

The author would like to thank Drs. M. Fukugita, L. McLerran, R. Pisarski, J. Sexton and A. Ukawa for discussions. He also expresses his deep thanks to the Theory Group at Fermilab for their warm hospitality.

References

[1] J. Kogut *et al.*, Phys. Rev. Lett. **50**, 393 (1983); J. Polonyi *et al.*, Phys. Rev. Lett. **53**, 644(1984).

[2] M. Fukugita and A. Ukawa, "Deconfining and Chiral Transitions of Finite-Temperature Quantum Chromodynamics in the Presence of Dynamical Quark Loops", RIFP-642, Kyoto, January, 1986.

[3] D. J. Gross, R. D. Pisarski and L. G. Yaffee, Rev. Mod. Phys. **53**, 43 (1981).

[4] B. Svetitsy and L. Yaffee, Nucl. Phys. **B210** [$FS6$], 423 (1981).

[5] R. D. Pisarski and F. Wilczek, Phys. Rev. **D29**, 338 (1984).

[6] T. A. DeGrand and C. E. DeTar, Nucl. Phys. **B225** [FS9], 590 (1983).

[7] T. Banks and A. Ukawa, Nucl. Phys. **B225** [FS9], 145(1983);

[8] J. Kogut *et al.*, Nucl. Phys. **B225**, 393 (1983).

[9] N. Weiss, Phys. Rev. **D25**, 2667 (1982).

[10] P. H. Damgaad, N. Kawamoto and K. Shigemoto, Nucl. Phys. **B264**, 1, (1986).

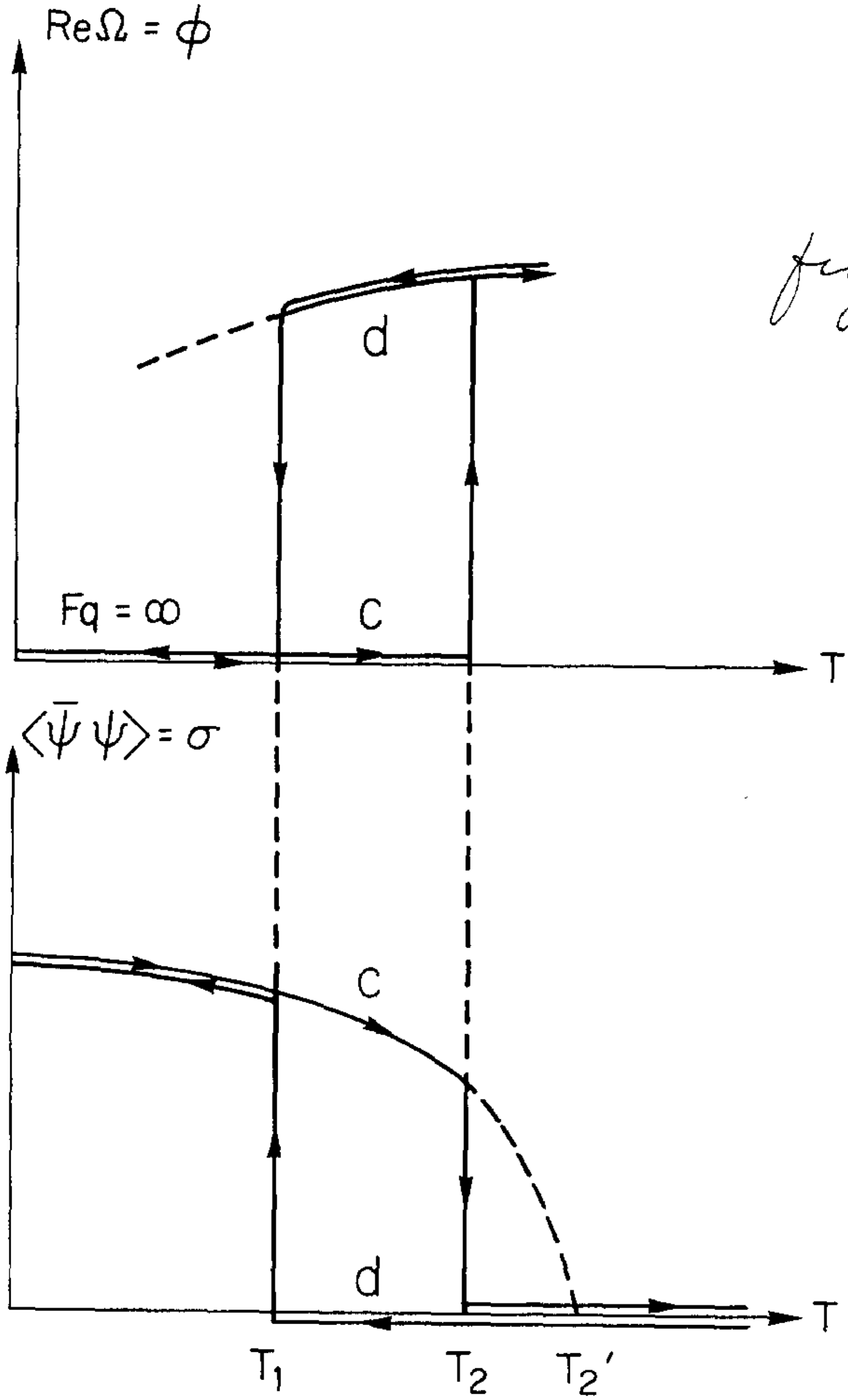
[11] K. Kajantie, C. Montonen and E. Pietarinen, Z. Phys. **C9**, 253 (1981).

[12] S. Midorikawa, M. So and S. Yoshimoto, "Renormalization Group approach to QCD Phase Transitions" INS-580 Tokyo, April, 1986.

Figure Captions

Figure 1: Schematic description of hysteresis curves motivated from the numerical computation. [2]

Figure 2: Stable and metastable states for three temperature regions.



$Re\Omega = \phi$
d
$F_q = \infty$
c
T
$\langle\bar{\psi}\psi\rangle = \sigma$
c
d
T
T_1
T_2
T_2'

fig 1

fig 2